THE GLAUBER MODEL AND THE HEAVY ION REACTION CROSS SECTION

Prashant Shukla

Nuclear Physics Division

Bhabha Atomic Research Center, Mumbai 400 085, India

Abstract

We reexamine the Glauber model and calculate the total reaction cross section as a function of energy in the low and intermediate energy range, where many of the corrections in the model, are effective. The most significant effect in this energy range is by the modification of the trajectory due to the Coulomb field. The modification in the trajectory due to nuclear field is also taken into account in a self consistent way. The energy ranges in which particular corrections are effective, are quantified and it is found that when the center of mass energy of the system becomes 30 times the Coulomb barrier, none of the trajectory modification to the Glauber model is really required. The reaction cross sections for light and heavy systems, right from near coulomb barrier to intermediate energies have been calculated. The exact nuclear densities and free nucleon-nucleon (NN) cross sections have been used in the calculations. The center of mass correction which is important for light systems, has also been taken into account. There is an excellent agreement between the calculations with the modified Glauber model and the experimental data. This suggests that the heavy ion reactions in this energy range can be explained by the Glauber model in terms of free NN cross sections without incorporating any medium modification.

I. INTRODUCTION

The Glauber model (GM) [1] has been employed for describing heavy ion reactions at high energies. It is a semiclassical model picturing the nuclei moving in a straight path along the collision direction, and gives the nucleus-nucleus interaction [2] in terms of interaction between the constituent nucleons (NN cross section) and nuclear density distributions. It is a well established model for high energies and has been applied to heavy ion collision for describing a number of reaction processes (See e.g. [3–5]). One of the most important physical quantities characterizing the nuclear reactions is the total reaction cross section. It is very useful for extracting information about the nuclear sizes. The Glauber model has been successively used to get the radii of radioactive nuclei from measured total reaction cross sections [6].

At low energies, the straight line trajectory is modified due to the Coulomb field between two nuclei. The Coulomb modified Glauber model (CMGM) [7,8] consists of replacing the eikonal trajectory at an impact parameter b, with the eikonal trajectory at the corresponding distance r_c of closest approach in the presence of the Coulomb field. Later the non eikonal nature of the trajectory around r_c, has also been taken into account [9]. Replacing r_c by the distance r_{cn} of closest approach in the presence of both the Coulomb and the nuclear field gives the Coulomb plus Nuclear modified Glauber model (CNMGM) [9,10]. This model (CMGM/CNMGM) has been widely used in recent literature [11–14]. Warner et. al. [11] have shown in their calculations for light projectiles on various targets that trajectory modifications are minor. Let us remark that the energies considered by them is much above the Coulomb barrier. The Coulomb modified Glauber model has also been used at very high energies [12] where trajectory modifications may be ineffective. There have been various prescriptions [13,14] to modify the NN cross sections due to nuclear density. Most of the work reported earlier [7,8,10,9,13,14] has been done using Gaussian densities. The reaction cross section is sensitive to surface density of the colliding nuclei. Depending on how well one has produced the surface texture with the Gaussian form, it will make a 5 to 10 %

change in the reaction cross section over that done with realistic densities [15].

In the light of this, we reexamine the various trajectory corrections in the Glauber model and calculate the total reaction cross section as a function of energy upto 50 times the Coulomb barrier. We quantify the energy range in which a particular correction is effective and find that when the center of mass energy of the system becomes 30 times the Coulomb barrier, the above modifications to GM are insignificant. Thus, the energy at and above which the results of CNMGM and GM coincide depends on the Coulomb barrier between the two nuclei and will be different for a light and heavy system. In the present work we use exact nuclear densities and free NN cross sections. The center of mass correction which is important for light systems has also been taken into account. Comparison of the experimental data with our calculations are presented in the plots of $\sigma_R(E)$ vs. E.

II. THE GLAUBER MODEL

Consider the collision of a projectile nucleus A on a target nucleus B. The probability for occurrence of a nucleon-nucleon collision when the nuclei A and B collide at an impact parameter $\mathbf{b}$ relative to each other is given by [4,5]

$$T(b)\bar{\sigma}_{NN} = \int \rho_A^z(\mathbf{b}_A)d\mathbf{b}_A\, \rho_B^z(\mathbf{b}_B)d\mathbf{b}_B\, t(\mathbf{b} - \mathbf{b_A} + \mathbf{b_B})\ \bar{\sigma}_{NN}. \tag{1}$$

Here $\rho_A^z(\mathbf{b}_A)$ and $\rho_B^z(\mathbf{b}_B)$ are the z-integrated densities of projectile and target nuclei respectively. $t(\mathbf{b})d\mathbf{b}$ is the probability for having a nucleon-nucleon collision within the transverse area element $d\mathbf{b}$ when one nucleon approaches at an impact parameter $\mathbf{b}$ relative to another nucleon. All these distribution functions are normalized to one. Here $\bar{\sigma}_{NN}$ is the average total nucleon nucleon cross section.

There can be upto $A \times B$ collisions. The probability of occurrence of n collisions will be

$$P(n, b) = \binom{AB}{n}(1 - s)^n(s)^{AB-n}. \tag{2}$$

Here, $s = 1 - T(b)\bar{\sigma}_{NN}$. The total probability for the occurrence of at least one NN collision in the collision of A and B at an impact parameter $\mathbf{b}$ is

$$\frac{d\sigma_R}{db} = \sum_{n=1}^{AB} P(n,b) = 1 - s^{AB}. \tag{3}$$

The total reaction cross section σ_R can be written as

$$\sigma_R = 2\pi \int bdb \left(1 - s^{AB}\right). \tag{4}$$

From this one can read the scattering matrix as

$$|S(b)|^2 = s^{AB} = (1 - T(b)\bar{\sigma}_{NN})^{AB}. \tag{5}$$

In the optical limit, where a nucleon of the projectile undergoes only one collision in the target nucleus can be written as

$$|S(b)|^2 \simeq \exp(-T(b)\bar{\sigma}_{NN}AB). \tag{6}$$

The scattering matrix can be defined in terms of eikonal phase shift $\chi(b)$ as

$$S(b) = \exp\left(-i\chi(b)\right). \tag{7}$$

If we assume $\bar{\alpha}_{NN}$ to be the ratio of real to imaginary part of NN scattering amplitude, the eikonal phase shift can be obtained as

$$\chi(b) = \frac{1}{2}\bar{\sigma}_{NN}(\bar{\alpha}_{NN} + i)\, AB\; T(b). \tag{8}$$

Here $\bar{\alpha}_{NN}$ will not be directly used for calculating the reaction cross section but will come through the correction in the trajectory due to the nuclear field. Once we know the phase shift and thus the scattering matrix, we can calculate the reaction cross section and also the elastic scattering angular distribution. In the present work we restrict ourselves to the calculations of reaction cross section only.

In momentum space $T(b)$ is derived as

$$T(b) = \frac{1}{2\pi}\int J_0(qb)S_A(\mathbf{q})S_B(-\mathbf{q})f_{NN}(q)qdq. \tag{9}$$

Here $S_A(q)$ and $S_B(-q)$ are the Fourier transforms of the nuclear densities and $J_0(qb) = 1/2\pi \int \exp(-qb\cos\phi)d\phi$ is the cylindrical Bessel function of zeroth order. The function

$f_{NN}(q)$ is the Fourier transform of the profile function $t(\mathbf{b})$ and gives the q dependence of NN scattering amplitude. The profile function $t(\mathbf{b})$ for the NN scattering can be taken as delta function if the nucleons are point particles. In general it is taken as a Gaussian function of width r_0 as

$$t(\mathbf{b}) = \frac{\exp(-b^2/2r_0^2)}{2\pi r_0^2}. \tag{10}$$

Thus,

$$f_{NN}(q) = \exp(-r_0^2 q^2/2). \tag{11}$$

Here, r_0 is the range parameter and has a weak dependence on energy (see for discussions [12]). We use $r_0 = 0.6$ fm [8] in the energy range 2 to 200 MeV.

The average NN scattering parameter $\bar{\sigma}_{NN}$ is obtained in terms of pp cross section σ_{pp} and nn cross section σ_{nn}, the parameterized forms for which are available in Ref. [8]. For projectile energies below 10 MeV/nucleon, we use the prescription given in Ref. [8] for computing $\bar{\alpha}_{NN}$. For energies above 10 MeV/nucleon, we take the parameterized forms from the work of Ref. [20]. These forms produce the elastic scattering data for various systems satisfactorily.

A. Optical potential in the Glauber model

The usual Glauber optical limit phase shift function is given by

$$\chi(b) = -\frac{1}{\hbar v}\int\limits_{-\infty}^{\infty} V_N(\mathbf{r}) dz. \tag{12}$$

Comparing Eq. (12) with Eq. (8), the optical potential $V_N(r)$ in the momentum representation can be identified as,

$$\begin{aligned} V_N(\mathbf{r}) &= -\hbar v\ \bar{\sigma}_{NN}(\bar{\alpha}_{NN} + i)\ \frac{AB}{2(2\pi)^3}\int e^{-i\mathbf{q}.\mathbf{r}} S_A(\mathbf{q}) S_B(-\mathbf{q}) f_{NN}(\mathbf{q}) d^3q, \\ &= -\hbar v\ \bar{\sigma}_{NN}(\bar{\alpha}_{NN} + i)\ \frac{AB}{(2\pi)^2}\int j_0(qr) S_A(\mathbf{q}) S_B(-\mathbf{q}) f_{NN}(\mathbf{q}) q^2 dq. \end{aligned} \tag{13}$$

Here $j_0(qr)$ is the spherical Bessel function of zeroth order.

B. Trajectory modifications in the Glauber model

The basic assumption in the Glauber model is the description of the relative motion of the two nuclei in terms of straight line trajectory. For low energy heavy ion reactions, the straight line trajectory is assumed at the distance of closest approach r_c calculated under the influence of the Coulomb potentials for each impact parameter b as given by,

$$r_c = (\eta + \sqrt{\eta^2 + b^2k^2})/k, \tag{14}$$

which is a solution of the following equation without the nuclear potential $V_N(r)$

$$E - \frac{Z_P Z_T e^2}{r} - \frac{\hbar^2 k^2}{2\mu}\frac{b^2}{r^2} - \mathrm{Re}V_N(r) = 0. \tag{15}$$

Here $\eta = Z_P Z_T e^2/\hbar v$ is the dimensionless Sommerfeld parameter. The CMGM [7,8] consists of replacing the eikonal trajectory at an impact parameter b with the eikonal trajectory at the corresponding distance r_c of closest approach. The distance of closest approach r_{cn} in the presence of both the Coulomb and nuclear potential [9] is obtained by solving Eq. (15) numerically, where $V_N(r)$ has been derived from Eq. (13). By replacing b by r_{cn}, CNMGM is obtained. The non eikonal trajectory [9] around r_c in the presence of the Coulomb field is represented by $r^2 = r_c^2 + (C+1)z^2$, where the quantity C is given by

$$C = \frac{\eta}{kb^2} r_c. \tag{16}$$

Thus in the CMGM, $T(b)$ will be simply replaced by $T(r_c(b))/\sqrt{(C+1)}$.

C. The nuclear densities

The two parameter fermi (2pf) density is given by

$$\rho(r) = \frac{\rho_0}{1 + \exp(\frac{r-c}{d})}, \tag{17}$$

where $\rho_0 = 3/\left(4\pi c^3(1 + \frac{\pi^2 d^2}{c^2})\right)$. Thus, the momentum density can be derived [16]as

$$S(q) = \frac{8\pi\rho_0}{q^3}\frac{ze^{-z}}{1-e^{-2z}}\left(\sin x\frac{z(1+e^{-2z})}{1-e^{-2z}} - x\cos x\right). \tag{18}$$

Where $z = \pi dq$ and $x = cq$. Here d is the diffuseness and c, the half value radius in terms of rms radius R for the 2pf distribution is calculated by $c = (5/3R^2 - 7/3\pi^2 d^2 - 5/3r_p^2)^{1/2}$. Here r_p is the proton radius. Equation (9) can be solved numerically for this density and the overlap integral can be extracted. In the present work, the density parameters have been taken from the compilation of measured charge density distributions [17,18] and are tabulated in Table I.

For lighter nuclei such as ^{12}C and ^{16}O, the densities are given in the from of harmonic oscillator (HO) densities as

$$\rho(r) = \rho_0\left(1 + \alpha\frac{r^2}{R^2}\right)\exp(-\frac{r^2}{R^2}), \tag{19}$$

where $\rho_0 = 1/\left[1 + 1.5\alpha\right]$. The momentum density is given by

$$S(q) = \rho_0\left(1 + 1.5\alpha - 0.25\alpha q^2 R^2\right)\exp\left(-\frac{q^2R^2}{4}\right). \tag{20}$$

Center of mass correction:

For lighter nuclei such as ^{12}C and ^{16}O we also take into account the corrections due to Center of motion. Such a correction for harmonic oscillator wave functions is given in Ref. [19]. With this the corrected density will become

$$S(q) = \rho_0\left(1 + 1.5\alpha - 0.25\alpha q^2 R^2\right)\exp\left(-\frac{q^2R^2}{4}\right)\exp\left(\frac{q^2R^2}{4A}\right). \tag{21}$$

TABLE I. Density parameters for various nuclei used in the present work

A	Z	Density form	d/α fm	R fm
12	6	HO	1.082	1.692
16	8	HO	1.544	1.833
28	14	2pf	0.537	3.150
40	20	2pf	0.563	3.510
58	28	2pf	0.560	3.823
90	40	2pf	0.550	4.274
208	82	2pf	0.546	5.513

III. RESULTS AND DISCUSSIONS

The reaction cross sections for light and heavy systems right from near coulomb barrier to intermediate energies have been calculated. We calculate the total reaction cross section as a function of energy upto 50 times the Coulomb barrier. The data for various systems and their references are given in Table II, III and IV. The Coulomb barrier is calculated from expression $V_C = Z_P Z_T 1.44/(1.5(A^{1/3} + B^{1/3}))$. The reduced mass M is defined as $M = AB/(A + B)$.

TABLE II. Reaction cross section data for ^{12}C on various targets along with their references

System	$E_{\rm lab}/A$ MeV/nucleon	σ_R mb	Ref.
^{12}C + ^{12}C	9.33	1444.00	[21]
$V_C/M = 1.258$	15.00	1331.00	[22]
	25.00	1296.00	–”–
	30.00	1315.00	[23]
	35.00	1259.00	[22]
^{12}C + ^{40}Ca	15.0	2165.0	[22]
$V_C/M = 2.186$	25.0	2030.0	–”–
	30.0	2014.0	–”–
^{12}C + ^{90}Zr	10.0	2219.0	[24]
$V_C/M = 3.214$	15.0	2297.0	[22]
	25.0	2415.0	–”–
	35.0	2528.0	–”–
^{12}C + ^{208}Pb	8.0	1754.0	[22]
$V_C/M = 5.068$	15.0	2873.0	–”–
	25.0	3236.0	–”–
	35.0	3561.0	–”–

TABLE III. Reaction cross section data for ^{16}O on various targets along with their references

System	E_{lab}/A MeV/nucleon	σ_R mb	Ref.
^{16}O + ^{16}O	2.0	73.86	[25]
$V_C/M = 1.524$	3.0	1136.00	–"–
	4.0	1300.00	–"–
	5.0	1395.00	–"–
	9.0625	1650.00	[26]
	15.625	1664.00	–"–
	21.875	1639.00	–"–
	30.000	1655.00	–"–
^{16}O + ^{28}Si	2.0625	509.0	[27]
$V_C/M = 1.90$	2.3750	765.6	–"–
	3.1250	1341.0	–"–
	3.4375	1451.0	–"–
	4.125	1626.0	–"–
	5.0625	1777.0	–"–
	8.9000	2013.0	–"–
	13.45	2067.0	–"–
	94.00	1757.0	[10]

TABLE IV. Reaction cross section data for ^{16}O on various targets along with their references

System	E_{lab}/A MeV/nucleon	σ_R mb	Ref.
^{16}O + ^{58}Ni	2.50	48.2	[28]
$V_C/M = 2.683$	2.75	288.7	–"–
	3.00	539.4	–"–
	3.75	994.5	–"–
	4.375	1282.0	–"–
	5.00	1485.0	–"–
	6.25	1765.0	–"–
	7.50	1953.0	–"–
^{16}O+^{208}Pb	5.00	124.1	[29]
$V_C/M = 5.019$	5.50	566.6	–"–
	6.00	939.6	–"–
	6.50	1171.0	[30]
	8.093	2023.0	[24]
	12.00	2847.0	–"–
	19.54	3452.0	[31]
	94.00	3600.0	[32]

Figure (1) shows the reaction cross section for ^{16}O + ^{16}O system calculated with CMGM with and without center of mass (c.m.) correction. This correction reduces the reaction cross section for lighter systems. Figures (2) to (5) show the reaction cross section for ^{16}O on various targets as a function of center of mass energy divided by the Coulomb barrier, calculated with the Glauber model (GM), the Coulomb modified Glauber model (CMGM) and the Coulomb plus nuclear modified Glauber model (CMNGM) along with the data. Figures (5) to (9) show the reaction cross section for ^{12}C on various targets. The center of mass correction has been taken into account in all the calculations. It can been seen from all the figures that the most significant effect in this energy range, is by the trajectory modification due to the Coulomb field. It is very significant upto energies $6V_C$. The modification in the trajectory due to the nuclear field tries to bring the results closer to GM. We universally find that when the center of mass energy of the system becomes 30 times the Coulomb barrier, the calculations with the CNMGM match with GM within 2 to 3%. Thus, the energy at and above which the results of CNMGM and GM coincide depends on the Coulomb barrier between the two nuclei and not on the energy per nucleon of the projectile. Further, it is observed that there is an excellent agreement between the calculations from the modified Glauber model and the experimental data. In contrast to [13,14] the present study suggests that the heavy ion reactions in this energy range can be explained by the Glauber model in terms of free NN cross sections without incorporating medium modification.

There are higher order corrections [33,34] to the optical limit phase shift function $\chi(b)$. These corrections are important at higher b but tend to become smaller at larger b [34] and are less significant for total reaction cross section which depend mostly upon peripheral collisions. They may become important for differential cross sections away from forward direction (which probe collisions at smaller b), increasingly at higher energies. The Gaussian densities [34] which produce only surface textures of the nuclear density are not adequate and realistic densities are to be used (as done in [12]) to calculate these corrections.

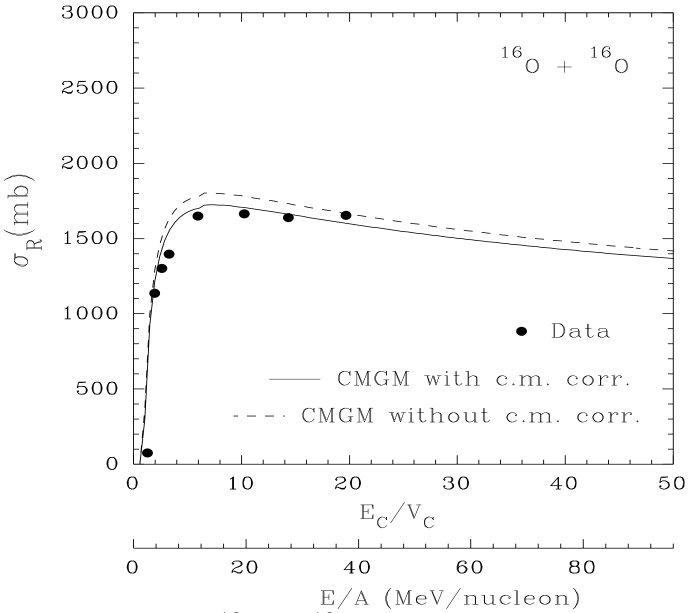

FIG. 1. Reaction cross section for ^{16}O + ^{16}O system calculated with the Coulomb modified Glauber model (CMGM) with and without center of mass (c.m.) correction.

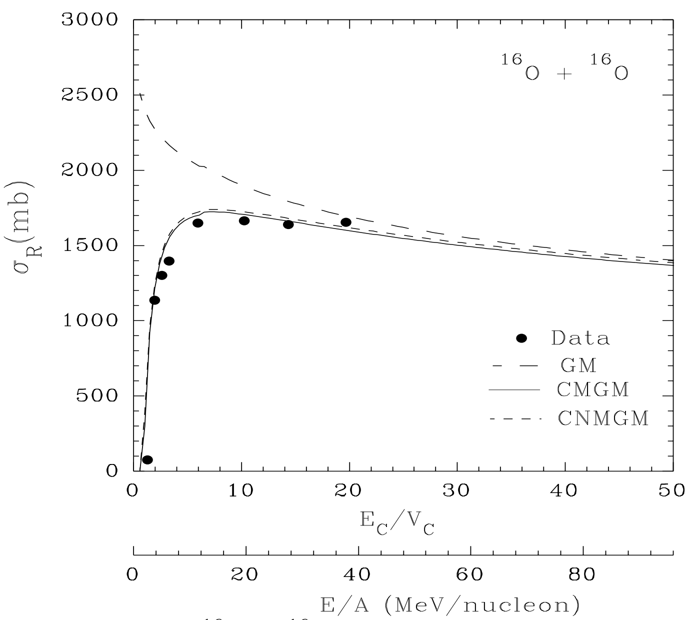

FIG. 2. Reaction cross section for ^{16}O + ^{16}O system calculated with the Glauber model (GM), the Coulomb modified Glauber model (CMGM) and the Coulomb plus nuclear modified Glauber model (CNMGM) along with the data.

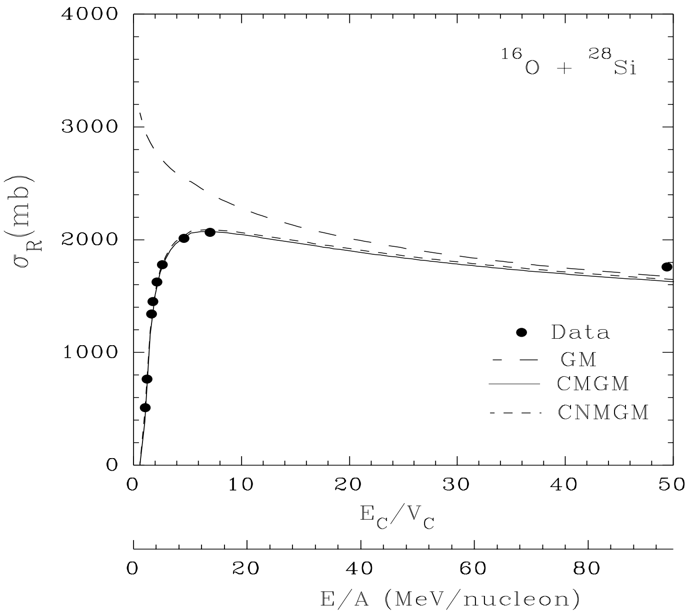

FIG. 3. Same as Fig. 2 but for ^{16}O + ^{28}Si system.

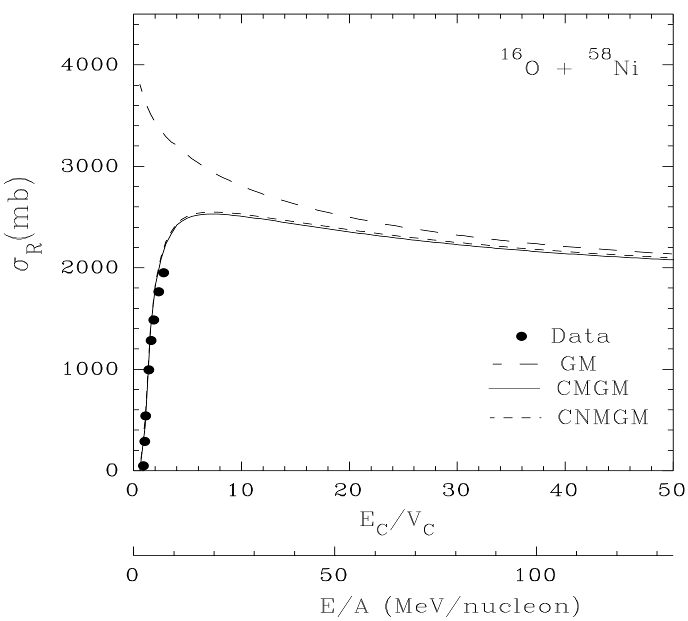

FIG. 4. Same as Fig. 2 but for ^{16}O + ^{58}Ni system.

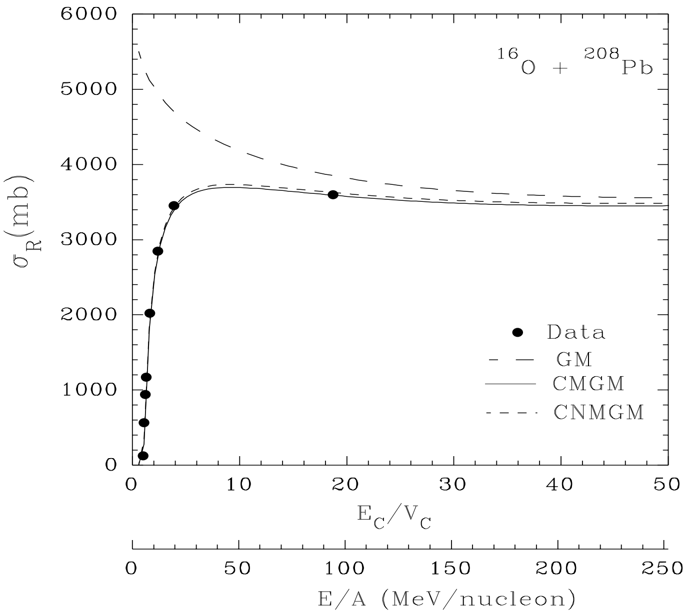

FIG. 5. Same as Fig. 2 but for ^{16}O + ^{208}Pb system.

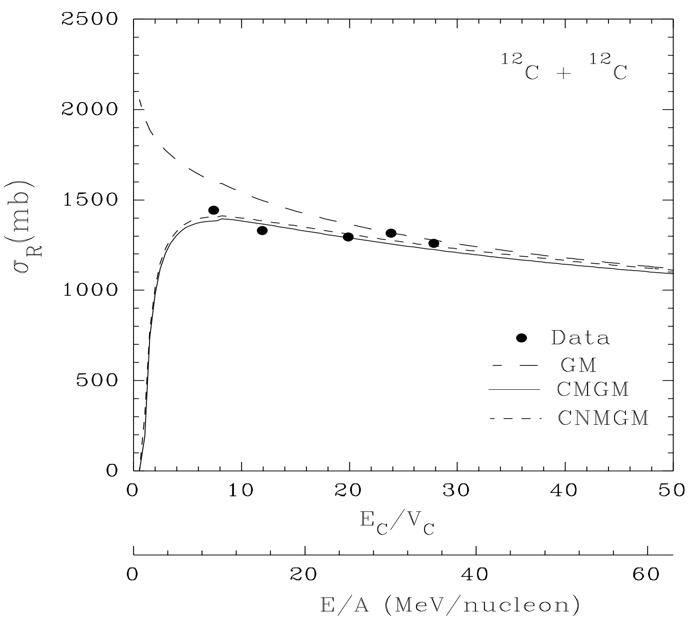

FIG. 6. Same as Fig. 2 but for ^{12}C + ^{12}C system.

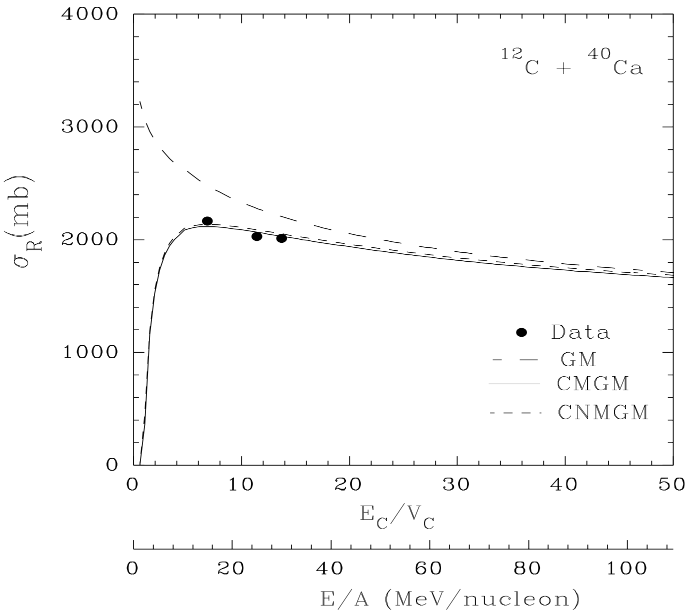

FIG. 7. Same as Fig. 2 but for ^{12}C + ^{40}Ca system.

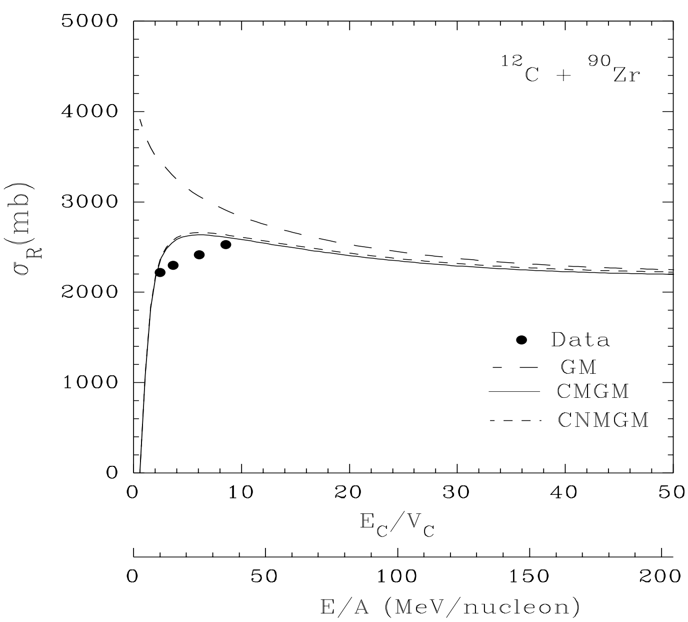

FIG. 8. Same as Fig. 2 but for ^{12}C + ^{90}Zr system.

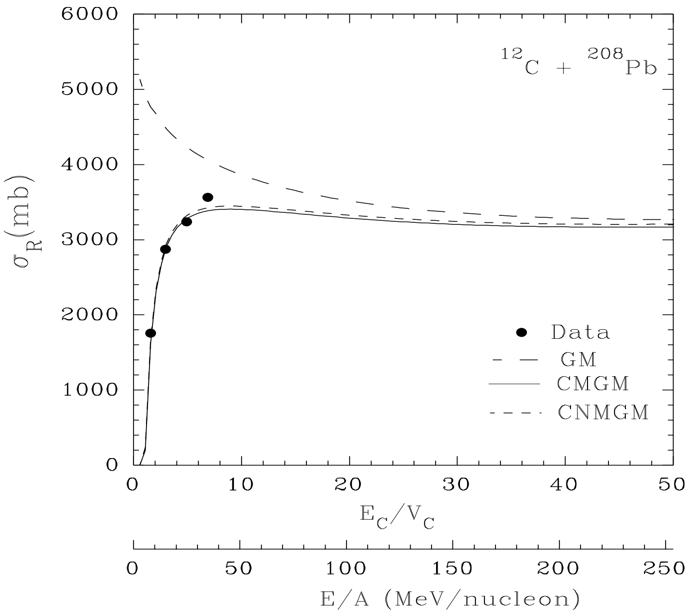

FIG. 9. Same as Fig. 2 but for ^{12}C + ^{208}Pb system.

IV. CONCLUSIONS

To summarize, we have reexamined the various trajectory corrections in the Glauber model and calculated the total reaction cross section as a function of energy upto 50 times the Coulomb barrier. The most significant effect in this energy range is by the trajectory modification due to the Coulomb field. The modification in the trajectory due to nuclear field is also taken into account. We derive $V_N(r)$ consistently from Eq. (13). We quantify the energy range in which a particular correction is effective and find that when the center of mass energy of the system is about 30 times the Coulomb barrier, no trajectory modification to GM is really required. Exact nuclear densities and free NN cross sections have been used in the calculation. The center of mass correction which is important for light systems has also been taken into account. There is an excellent agreement between our calculations including all the modifications discussed in the manuscript, and the experimental data. In contrast to [13,14] the present study suggests that the heavy ion reactions in this energy range can be explained by the Glauber model in terms of free NN cross sections without invoking any medium modification. It implies that the density effects on the NN cross sections are either absent or very minor. One possible explaination for this is the fact that for heavy ions, the contribution to the reaction cross section comes from the surface region where the density is very small.

ACKNOWLEDGMENTS

The author acknowledges the stimulating discussions with Z. Ahmed, S. Kailas, and S.V.S Sastry. He is also thankful to V. Jha for providing the reaction cross section data for some of the systems.

REFERENCES

[1] R.J. Glauber, *Lectures on theoretical Physics* (Inter-Science, New York, 1959) Vol. I.

[2] P.J. Karol, Phys. Rev. C**11**, 1203 (1975).

[3] Jean-Paul Blaizot and Jean-Yves Ollitrault, Physical Review Latters 77, 1703 (1996); S. R. Klein and J. Nystrand, Phys. Rev. C**60**, 014903 (1999).

[4] C.Y. Wong, *Introduction to High Energy Heavy Ion Collisions*, World Scientific, Singapore, 1994.

[5] P. Shukla, Preprint: nucl-th/0112039 (2001).

[6] I. Tanihata et al., Phys. Rev. Lett. **55**, 2676 (1985); I. Tanihata, Nuclear Physics **A488**, 113c (1988); I. Tanihata et. al, Phys. Lett. **B289**, 261 (1992).

[7] A. Vitturi and F. Zardi, Phys. Rev. C**36**, 1404 (1987); S.M. Lenzi, A. Vitturi, F. Zardi, Phys. Rev. C**40**, 2114 (1989).

[8] S.K. Charagi and S.K. Gupta, Phys. Rev. C**41**, 1610 (1990); Phys. Rev. C**46**, 1982 (1992).

[9] S. K. Gupta and P. Shukla, Phys. Rev. C**52**, 3212 (1995).

[10] M.H. Cha, Phys. Rev. C**46**, 1026 (1992).

[11] R. E. Warner, M. H. McKinnon, and H. Thirumurthy, and A. Nadasen, Phys. Rev. C**59**, 1215 (1999).

[12] I. Ahmad, M. A. Abdulmomen, and M. S. Al-Enazi, Phys. Rev. C**65**, 054607 (2002).

[13] Cai. Xiangzhou, Feng Jun, Shen Wenqing, Ma Yugang, Wang Jiansong, and Ye Wei, Phys. Rev. C**58**, 572 (1998).

[14] A. de Vismes, P. Roussel-Chomaz, and F. Carstoiu, Phys. Rev. C**62**, 064612 (2000).

[15] S.K. Charagi and S.K. Gupta, Phys. Rev. C**56**, 1171 (1997).

[16] S. K Gupta, Z. Phys. **A331** (1988) 457.

[17] C.W. de Jager, H. de Vries and C. de Vries, Atom. Nucl. Data Tables **14**, 479 (1974).

[18] H. de Vries, C.W. de Jager and C. de Vries, Atom. Nucl. Data Tables **36**, 495 (1987).

[19] V. Franco and A. Tekou, Phys. Rev. C**16**, 658 (1977).

[20] W. Grein, Nucl. Phys. **B131**, 255 (1977). S.K. Charagi, Ph.D. Thesis, University of Mumbai (1994).

[21] S. Kox et. al, Phys. Rev. C**35**, 1678 (1987).

[22] C. C. Sahm, et. al, Phys. Rev. C**34**, 2165 (1986).

[23] J. Chauvin, D. Lebrun, F. Durand and M. Buernerd, J. Phys. **G261**, 1985.

[24] J.B. Ball, C.B. Fulmer, E.E. Gross, M.L. Halbert, D.C. Hensley, C.A. Ludemann, M.J. Saltmarsh and G.R. Satchler, Nucl. Phys. **A 252**, 208 (1975).

[25] R. Bass, *Nuclear reactions with heavy ions*, (Springer-Verlag, NY), 1980.

[26] Dao T. Khoa, Phys. Rev. Lett. **74**, 34 (1995).

[27] J.G. Cramer, R.M. Devries, D.A. Goldberg, M.A. Zisman and C.F. Maguire, Phys. Rev. C**14**, 2158 (1976).

[28] N. Keeley et.al, Nucl. Phys. **A582**, 314 (1995).

[29] B.T. Kim, H.C. Kim and K.E. Park, Phys. Rev. C**998** (1988).

[30] F.D. Bechetti, Phys. Rev. C**6**, 2215 (1972).

[31] G. R. Satchler and W. G. Love, Phys. Rep. **55**, 183 (1979).

[32] P. Roussel-Chomaz et.al, Nucl. Phys. **A477**, 345 (1988).

[33] S. J. Wallace, Phy. Rev. C**12**, 179 (1975).

[34] V. Franco and G. K. Varma, Phys. Rev. C**15**, 1375 (1977).